DB long transfer line : optimisation of the optics
B. Jeanneret 18.07.07 at the CLIC beam dynamics meeting. Draft for discussion.

1 Optimising the lattice

Consider a FODO lattice

free parameters : $L = L_{\text{cell}}$ $N = N_{\text{cell}}$ $\mu = \mu_{\text{cell}}$ $\hat{\beta}$

Constraints : $L_0 = L_{\text{line}}$ $L, \mu, \hat{\beta}$: one fixed by the two others

$\rightarrow$ two free parameters

$\hat{\beta}$: to be fixed considering vacuum issues (in particular)

$\rightarrow$ express everything w.r.t to $\hat{\beta}$ and μ

Criteria :

- number of cells

$$N = \frac{L_0}{\hat{\beta}} \frac{1 + \sin\frac{\mu}{2}}{\sin\mu} \ . \tag{1}$$

- overall magnet power

$$A_p(\hat{\beta}, \mu) = \frac{4L_0}{\hat{\beta}} \frac{(1 + \sin\frac{\mu}{2})^3}{\sin\mu\cos^2\frac{\mu}{2}}. \tag{2}$$

- overall induced kick by quadrupole displacement

$$\frac{\Delta_x}{\sigma_\beta} = \frac{\Delta_x}{\epsilon\beta} = \frac{2(1 + \sin\frac{\mu}{2})}{\hat{\beta}\cos\frac{\mu}{2}} \sqrt{\frac{L_0}{\epsilon\sin\mu}} \delta_x \tag{3}$$

- chromaticity

$$C = -\frac{L_0}{2\pi\hat{\beta}} \frac{1 + \sin\frac{\mu}{2}}{\cos\frac{\mu}{2}} \tag{4}$$

A good simplification appears : All the functions are factorised,

$$f(\hat{\beta}, \mu) = g(\hat{\beta})\, h(\mu) \tag{5}$$

KICKS :

$$\delta_x = 100\mu\,\text{m} \ \rightarrow \ \frac{\Delta_{\text{x}}}{\sigma_\beta} = 3.5 \tag{6}$$

Avoiding filamentation with δ_p :

- Either implement a chromatic correction
- Or use sliding bumps

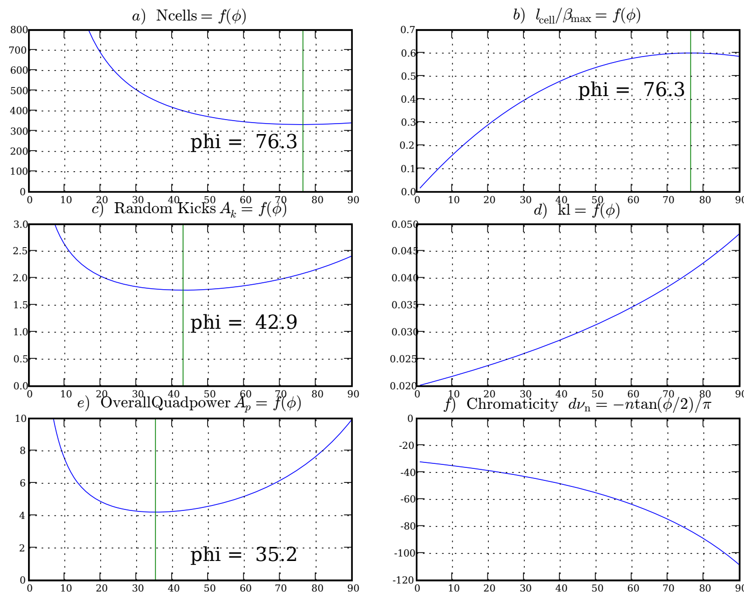

Figure 1: Some functions to optimise a functions of the cell phase advance at fixed $\hat{\beta} = 100m$.

mu	Ncell	Ncell rel.	pow [a.u.]	kick [a.u.]	$d\nu$ [2π] at $\delta_p = 0.02$
5	2395	7.19	3.10	1.99	-0.67
10	1252	3.76	1.77	1.47	-0.70
15	874	2.62	1.35	1.26	-0.73
20	686	2.06	1.16	1.15	-0.77
25	576	1.73	1.06	1.08	-0.81
30	504	1.51	1.01	1.04	-0.86
35	454	1.36	1.00	1.01	-0.91
40	418	1.25	1.01	1.00	-0.97
45	391	1.17	1.04	1.00	-1.03
50	371	1.12	1.08	1.01	-1.10
55	357	1.07	1.15	1.02	-1.18
60	346	1.04	1.23	1.05	-1.27
65	339	1.02	1.34	1.08	-1.38
70	335	1.01	1.47	1.11	-1.49
75	333	1.00	1.62	1.16	-1.63
80	334	1.00	1.82	1.22	-1.78
85	336	1.01	2.06	1.28	-1.96
90	341	1.03	2.36	1.36	-2.17

2 Notations

Cell length	L	m
Transfer line length	L_0	m
Maximum of beta function	$\hat{\beta}$	m
Minimum of beta function	$\check{\beta}$	m
Phase advance per cell	μ	rad
Total phase advance of the line	ν	rad/2π
Number of cell of the line	N	-
Focal length of the quadrupoles	f	m
Integrated gradient of the quadrupoles	$kl = 1/f$	m^{-1}
Emmitence at 2.5 Gev/c	$\epsilon = 2.04.e - 8$	m×rad

Auxiliary relations :

$$\sin\mu = 2\sin\frac{\mu}{2}\cos\frac{\mu}{2} \tag{7}$$

3 FODO basic

The transfer matrix from the middle of a F-quadrupole to the middle of the next F-quadrupole is

$$M_{\rm FF} = \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ \gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix} = \begin{pmatrix} 1 - \frac{L^2}{8f^2} & L(1+\frac{L}{4f}) \\ -\frac{L}{4f^2}(1-\frac{L}{4f}) & 1-\frac{L^2}{8f^2} \end{pmatrix} \tag{8}$$

With μ variable and $m_{11} = m_{22} \Rightarrow \ \alpha = 0$, so :

$$\cos\mu = 1 - \frac{L^2}{8f^2} \tag{9}$$

and

$$\sin\frac{\mu}{2} = \frac{L}{4f} = \frac{(kl)L}{4} \ . \tag{10}$$

Writing $\hat\beta = \beta$ and substituting (10) in m_{12}, we get

$$\hat\beta = L\frac{1+\sin\frac{\mu}{2}}{\sin\mu} \ . \tag{11}$$

The matrix $M_{\rm DD}$ is obtained by replacing $f \to -f$ in $M_{\rm FF}$. It follows that $m_{11} = m_{22}$ are unchanged and μ is still given by (10). So finally

$$\check\beta = L\frac{1-\sin\frac{\mu}{2}}{\sin\mu} \ . \tag{12}$$

The number of cells is $N = L_0/L$. Using (11) for L, it follows

$$N = \frac{L_0}{L} = \frac{L_0}{\hat\beta}\frac{1+\sin\frac{\mu}{2}}{\sin\mu} \ . \tag{13}$$

The quadrupole strength can be expressed with using (10) and (11) :

$$kl = \frac{1}{f} = \frac{4}{\hat\beta}\frac{\sin\frac{\mu}{2}(1+\sin\frac{\mu}{2})}{\sin\mu} \tag{14}$$

4 Chromaticity

$$C_{\rm cell} = -\frac{1}{4\pi}\int_{\rm cell}\beta(s)k(s)ds = -\frac{1}{4\pi}(\hat\beta - \check\beta)\times(kl) = -\frac{1}{4\pi}\frac{\hat\beta-\check\beta}{f} \tag{15}$$

Using (10), (11,12) and (7), we get

$$C_{\rm cell} = -\frac{\tan\frac{\mu}{2}}{\pi} \tag{16}$$

For N cells, $C = NC_{\rm cell}$. With (13) and (7),

$$C = -\frac{L_0}{2\pi\hat\beta}\frac{1+\sin\frac{\mu}{2}}{\cos\frac{\mu}{2}} \tag{17}$$

5 Parasitic kicks

The displacement of the beam at the end of the line ($s = L_0, \beta(L_0) = \beta$), which results from random kicks associated to a r.m.s. displacement $\delta x'$ of the qudrupoles is

$$\Delta = \sum_{\mathrm{i}}^{\mathrm{2N}} \sqrt{\beta\beta_i} \sin\phi_i \, \delta x' \quad \text{with} \quad \phi_i = N\mu - i\frac{\mu}{2} \ . \tag{18}$$

With $< \sin^2\phi_i >= \frac{1}{2}$ and $\delta x' = (kl)\delta x$, the average of the quadratic displacement is

$$< \Delta^2 >= \sum_{\mathrm{i}}^{\mathrm{2N}} < \Delta_i^2 >= \frac{1}{2} \sum_{\mathrm{i}}^{\mathrm{2N}} \beta\beta_i (kl)^2 < \delta x^2 > \ . \tag{19}$$

A figure of merit for parasitic kicks can be written :

$$A_k^2 = \frac{< \Delta^2 >}{\beta < \delta x^2 >} = \frac{1}{2} \sum_{\mathrm{i}}^{\mathrm{2N}} (\hat{\beta} + \check{\beta})(kl)^2 = \frac{N}{2} (\hat{\beta} + \check{\beta})(kl)^2 \, . \tag{20}$$

With N from (13), kl from (10) and $\hat{\beta} + \check{\beta} = 2L/\sin\mu$ from (11,12)

$$A_k^2 = \frac{16L_0}{L^2} \frac{\sin^2\frac{\mu}{2}}{\sin\mu} \tag{21}$$

Finally with L from (11) and (7),

$$A_k^2 = \frac{14L_0}{\hat{\beta}^2} \frac{(1 + \sin\frac{\mu}{2})^2}{\sin\mu\cos^2\frac{\mu}{2}} \tag{22}$$

Writing $\Delta_x = \sqrt{< \Delta^2 >}$ and $\delta_x = \sqrt{< \delta x^2 >}$, the normalised quadratic displacement writes

$$\frac{\Delta_x}{\sigma_\beta} = \frac{\Delta_x}{\epsilon\beta} = \frac{2(1 + \sin\frac{\mu}{2})}{\hat{\beta}\cos\frac{\mu}{2}} \sqrt{\frac{L_0}{\epsilon\sin\mu}} \delta_x \tag{23}$$

6 Installed power

Considering initially the case $\hat{\beta}$ =constant,i.e constant quadrupole aperture, then the current $I \sim kl$ and the overall needed power to feed the quadrupoles is $P \sim NI^2 \sim N(kl)^2$. The figure of merit writes

$$A_p = N\,(kl)^2 \ . \tag{24}$$

Using (10) for kl and $N = L_0/l$, we get

$$A_p = 16\frac{L_0 \sin^2\frac{\mu}{2}}{L^3}. \tag{25}$$

Substituting L from (11) and using (7), we obtain

$$A_p(\mu) = \frac{4L_0}{\hat{\beta}^3}\frac{(1+\sin\frac{\mu}{2})^3}{\sin\mu\cos^2\frac{\mu}{2}}. \tag{26}$$

Then if $\hat{\beta}$ varies the aperture of the quadrupoles will be $a = n\sigma_\beta \sim \hat{\beta}^{1/2}$. With fixed kl, $P \sim I^2 \sim a^4 \sim \hat{\beta}^2$. It follows

$$A_p(\hat{\beta},\mu) = \frac{4L_0}{\hat{\beta}}\frac{(1+\sin\frac{\mu}{2})^3}{\sin\mu\cos^2\frac{\mu}{2}}. \tag{27}$$

7 Length and number of cells

The quantity $L/\hat{\beta}$ can be used to optimize the number of cells N. With fixed $\hat{\beta}$. Using (11),

$$\frac{L}{\hat{\beta}} = \frac{\sin\mu}{1+\sin\frac{\mu}{2}} \tag{28}$$

while (13) gave

$$N = \frac{L_0}{\hat{\beta}}\frac{1+\sin\frac{\mu}{2}}{\sin\mu} \ . \tag{29}$$

Disregarding the constant factors, the two quantities are inverse of each other as it shall be.